Proceedings of the Annual General Donald R. Keith Memorial Conference
West Point, New York, USA
May 2, 2019
A Regional Conference of the Society for Industrial and Systems Engineering

Stochastic Value Modeling: Clarity of Action in Complex Decision Making

Brandon Shively and Patrick DuBois

Department of Systems Engineering
United States Military Academy, West Point, NY

Corresponding author: brandon.shively@westpoint.edu

Author Note: Cadet Brandon Shively is a Systems Engineering Major at the United States Military Academy (USMA) at West Point. His interest in the topics covered in this thesis stems from his experiences in modeling and decision-making classes in the Systems Engineering Department at West Point. Major Patrick DuBois is part of West Point's Department of Systems Engineering. MAJ DuBois instructs classes at the academy on both deterministic and stochastic modeling and advised Cadet Shively in the modeling and research for this thesis. Any questions can be sent to the Cadet Shively.

Abstract: The Department of System's Engineering at West Point recently developed a new quantitative method for a post-simulation comparison of stochastically assessed alternatives. They applied this method to a stochastic value model, comparing six alternatives based on their uncertain cost and value. This independent study will evaluate and analyze this new method to determine if it has significantly differing results from traditional value models and analyses. This study takes results from a traditional value model and compares them to results of the stochastic method used by the Department of Systems Engineering at West Point. This study specifically uses in a car-buying experience as its base of analysis and also includes applications to more complex decisions. Using these case studies, this analysis intends to prove or disprove the progressive impact that stochastic modeling can have on complex decision making and reveal how useful this new form of analysis is to modeling and decision-making processes.

Keywords: Decision-Making under Uncertainty, Analysis of Alternatives (AoA), Stochastic Modeling, Buyers Confidence

1. Introduction

In everyday life people and large corporations are faced with high-risk, complex decisions that involve uncertainty. One common method to handle such decisions is through value-based modeling. Using this method, alternatives are decomposed into scorable categories known as value measures. Each of these value measures are given a factor of importance to the decision maker (DM). The alternatives are scored in each category and compared to one another based on their total value and cost scores. (Parnell, G. S., Driscoll, P. J., & Henderson, D. L., 2011).

The utility of traditional value modeling is well documented (Loerch, A. G., Rainey, L. B., 2007), but aside from sensitivity analysis, lacks a robust treatment of uncertainty. Professor Ron Howard states "The most challenging phenomenon we face in decision making is uncertainty" (Howard, R., & Abbas, A., 2016). Ignoring the inherent uncertainty in complex decisions results in deceptively crisp cost and total value scores that may rob the DM of potentially critical information.

To achieve more realistic results, the value models must account for this uncertainty in the value and cost measures. Researchers within the Department of Systems Engineering at West Point continue to explore this field. This paper provides a review of their methodology and poses the hypothesis that stochastic value modeling (SVM) and analysis provides the DM with more comprehensive results than the results from a purely traditional approach, leading to more informed decision making and ultimately clarity of action.

The remainder of this paper will: (1) Provide a literature review on the modeling of uncertainty showing the significance of SVM and follow-on levels of analysis. (2) Fully explain the two levels of analysis created by the researchers within the Department of Systems Engineering at West Point. (3) Discuss the methodology used to evaluate these analyses in their ability to inform decisions and create clarity of action for the DM. (4) Apply these new techniques to real-life decisions and explain the outcomes. (5) Finally, the conclusions and results of the proposed thesis are outlined.

1.1 Literature Review

DMs have always struggled to attain clarity of action because our futures are uncertain; this uncertainty is what makes decision making so difficult (Howard, R., & Abbas, A., 2016). The integration of uncertainty into decision science has been studied for decades in attempts to provide more realistic information to DMs. The reason this has proven to be so difficult is that it is hard to define uncertainty. When looking only at data, there are two types of uncertainty - uncertainty created from incomplete information and uncertainty due to inaccurate information (Lee, M., & Dry, M., 2006). Packard, Clark, and Klein (2017) explain that there are four fundamental domains of uncertainty that are based off the "openness" or "closedness" of the options and outcomes; risk and ambiguity, environmental uncertainty, creative uncertainty, and absolute uncertainty. They explain that "*true uncertainty*" is the objective conditions of the environment and how they interact with the DM's perceptions. This notion of *true uncertainty* alludes to a closed set of alternatives where the number of options is constant. This demonstrates that when modeling and analyzing uncertainty for DMs, risk and ambiguity, and environmental uncertainty are the types of uncertainty that are being reduced. These can collectively be noted as *true uncertainty*

There are many methods to model *true uncertainty* in decision making so that DMs can make more informed decisions. Value modeling traditionally employs sensitivity analysis on the importance (known as weights) associated with value measures (Parnell, G. S., Driscoll, P. J., & Henderson, D. L., 2011). However, it was found that these computed weight vectors were usually assumed to be constant causing irrationalities in the results (Liu, S., Yu, W., Liu, L., & Hu, Y., 2019). In reality, these weight vectors are unknown, ambiguous values that change due to the perspectives of the DM. Another technique used to include uncertainty in decision making is through "Intuitional Fuzzy Sets" (IFS). This approach is helpful in complex decision-making because it "is utilized when input values and parameters are subjective and vague" (Mousavi, S., Jolai, F., & Tavakkoli-Moghaddam, R., 2011), allowing for much of the *true uncertainty* to be captured. The primary drawback of this method, however, is that it uses a complex statistical approach that is difficult to understand for some DMs.

Another more applicable method is stochastic multi-attribute decision making "that defines probability distributions for each input value or parameter in the decision-making process," (Mousavi, S., Jolai, F., & Tavakkoli-Moghaddam, R., 2011). The reason this method is so valuable is that when representing an unknown value in a model, it is more appropriate to represent it as a distribution (Savage, S., 2012). A distribution enables the model to represent an unknown value by possible outcomes rather than a representative statistic such as its average. This method of integrating *true uncertainty* in modeling has proven very beneficial in assisting with decision making, especially when the decision involves random variability in the parameters (Mousavi, S., Jolai, F., & Tavakkoli-Moghaddam, R., 2011). This method allows for analysis of the results through stochastic pareto charts, cumulative distribution function (CDF) charts, and tornado charts (MacCalman, A., & Parnell, G., 2016). The use of stochastic pareto charts and CDFs is seen in the work *Selecting Robotic Power Solutions: A Case Study of Stochastic Value Modeling* (Mittal, V., Lesinski, G., & MacCalman, A., 2017). Mittal et al. leverage these techniques to choose a power source for a robotic application. Their study focused on the analysis of a stochastic pareto chart as well as a stochastic dominance test on the created CDFs. Their analysis found that a partially dominated solution could end up being a dominating solution. This novel result shows the vital application of SVM and its need to be extrapolated upon.

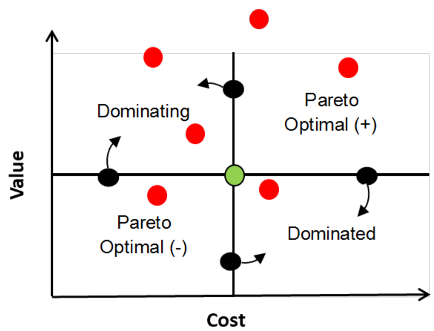
Although this form of SVM has proven to aid DMs make more informed decisions, there are further analyses that can be conducted to generate even more clarity of action. After the release of *Selecting Robotic Power Solutions: A Case Study of Stochastic Value Modeling,* the USMA's Operations Research Center created two follow-on levels of analysis that utilize the pareto charts to conduct a more in-depth analysis for the DM. These two methods of analysis are documented in two presentations titled: *Unmasking Uncertainty in Multi-Attribute Value Models (Level 1): Exploring Uncertainty on the Frontier* and *Unmasking Uncertainty in Multi-Attribute Value Models (Level 2): Illuminating the Trades we Expect* (Caddell, J., DuBois, P., Driscoll P., & Dabkowski, M., 2018). The techniques utilized in these two levels of analysis will be summarized, applied, and analyzed for practicality in generating clarity of action for the DM.

1.2 Background of Analyses Methods

The Department of Systems Engineering's method for a post-simulation comparison of stochastically assessed alternatives builds on the work completed by Mittal et al. It proposes a method for quantitatively assessing simulated alternatives using simple analytic method intended to be easily understood and assessible to DMs without an analytic background. The method is broken in two distinct steps identified as Level 1 and Level 2 analysis.

1.2.1 Level 1 Analysis.

The first iteration, dubbed the "Level 1 analysis," leverages the concept of dominance and pareto optimality. Figure 1 provides the labels. The first classification is known as "dominating," in which the selected point has a higher value while maintaining a lower cost than the other alternative. This is where the DM will be most happy. The second is "pareto optimal (+)," in which both cost and value of the selected point is larger than that of the other alternative. Third is "pareto optimal (-)," in which both cost and value of the selected point is smaller than the other alternatives. The last is "dominated," in which the selected point has a higher cost and a lower value. The method compares every simulated cost-value outcome of one alternative to every possible outcome of another alternative and tracks the comparisons. Reference Figures 1 and 2 below. When comparing the green alternative to the red alternative, select a single simulated cost-value outcome of the green alternative – that point becomes the new origin. Compare that point to every simulated red outcome. Count the number of points that fall in each quadrant. Select another simulated green outcome and execute the process again. Continue until every green dot is compared to every red dot. A sample output for the first iteration is provided in Figure 2. (DuBois 2018)



	Count	% of Total
Red dominates	2	33%
Green dominates	1	17%
Red: Pareto Optimal (+)	2	33%
Red: Pareto Optimal (-)	1	17%

Figure 1: Level 1 Analysis (DuBois 2018)   Figure 2: Sample Level 1 Analysis Output

When compared with the raw data, the aggregated results provide a clear indication of what could happen, and which alternative is mostly likely to provide the best combination of cost and value.

1.2.2 Level 2 Analysis.

If the DM needs more information before committing to an alternative or deciding between two predominantly pareto optimal alternatives, the Level 2 analysis increases the granularity of the analysis in the pareto optimal region. The three zones within the pareto optimal (+) region are found by drawing a line from the expected cost-value point of one alternative (shown in green below) through the expected cost-value point of the other alternative (in this case red). Zone 2 encompasses all points with a higher cost and value, but also has a better cost-value tradeoff than exists between the averages of the two alternatives' cost and value. This paper refers to this line as the "average's trendline." Create Zones 3 and 4 by asking the DM how much more could Red cost, at the same value level before they'd no longer select that alternative. This line is unique and is coined the "comfortability trendline." Zone 3 then includes all possible realizations of Red that are acceptable, and Zone 4 contains all realizations of Red that would be considered unacceptable. (Caddell 2018)

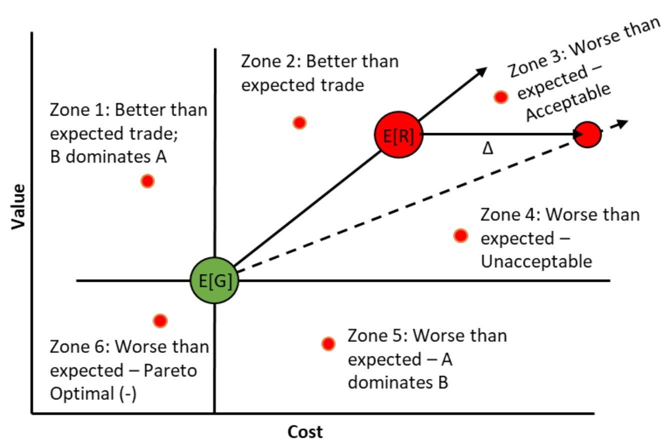


Figure 3: Level 2 Analysis (adapted from Caddell 2018)

The output of a Level 2 analysis (Figure 3) looks similar to that of a Level 1 analysis with the addition of a couple extra zones. It provides the decision with a more nuanced assessment of the likely realization of selecting an alternative relative to their comfort level – in other words, how much risk is associated with selecting a particular alternative.

ISBN: 97819384961-6-5

2. Methodology

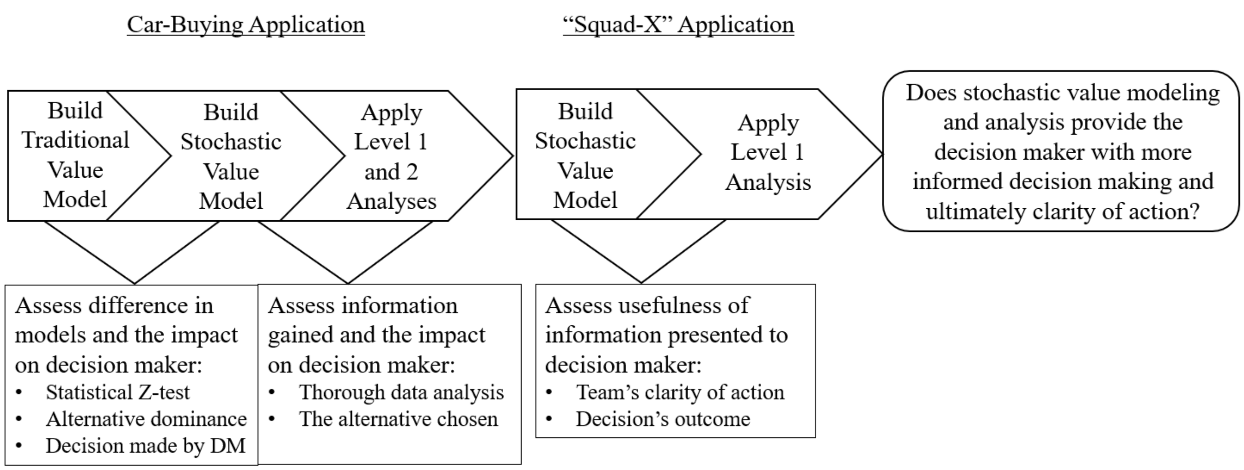


Figure 4: Methodology to Assess the Impact of SVM and Level 1 and 2 Analyses on DMs

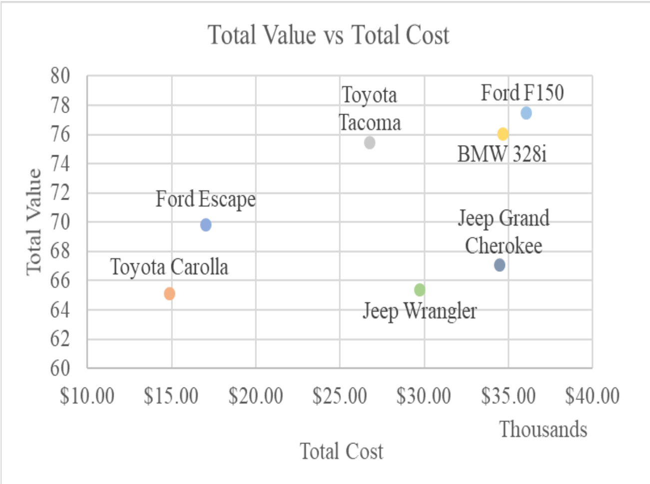
This paper follows an application method of assessment, as seen in Figure 4. To analyze the impact of the Level 1 and 2 analyses in aiding DMs make informed decisions this paper applies the technique to a common decision - buying a car. The second method of assessment is an application of the Level 1 analysis to a current USMA undergraduate engineering capstone project called "Squad-X Micro-Breaching." Both applications were applied to real stakeholders' decisions and observed to see if the DM's experience is positive, negative, or indifferent.

3. Application and Analysis

To assess the impact of SVM and Level 1 and 2 analyses on improving clarity of action for DMs, these techniques were applied to two decisions – buying a car and a USMA capstone project titled "Squad-X Micro-Breaching." This section of the paper will: (1) Explain the application and results of a traditional and stochastic value model to helping cadets choose a car to buy. (2) Explain the application and results of Level 1 and 2 analyses to a case study of one cadet's experience in buying a car. (3) Finally, ddiscuss the application and results of a stochastic model and Level 1 analysis to a USMA capstone project that worked with DARPA.

3.1 Application to Buying a Car

These decision aiding techniques were first applied in choosing a car to buy. This was done by assisting USMA cadets in the process of buying a car, most of whom had narrowed it down to a few alternatives. These value models use seven value measures to model seven car alternatives with which the cadets were interested. The raw cost and performance data was supplied from US News' used-car rankings and data (*How We Rank Used Cars.* n.d.). The data was modeled stochastically (represented as distributions) and displayed in Figure 6 using the SIP Math Microsoft Excel add-in (*Sipmath.* n.d). The car-buying value model produced results found in Figure 5 and the stochastic value model produced results found in Figure 6.



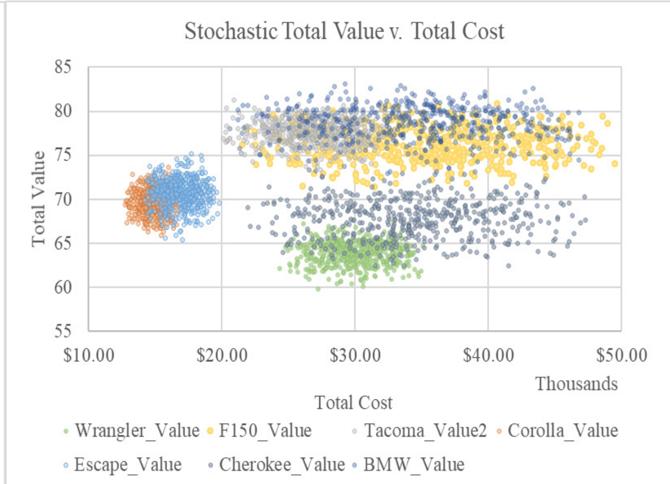


Figure 5: Traditional Car Alternative Value v. Cost

Figure 6: Stochastic Car Alternative Value v. Cost

The first step in the assessment is to compare the two models' cost versus value graphs. When visually comparing the alternatives in Figure 5, the cadets witnessed that the only alternatives completely dominated are the Jeeps, while the rest lie along the efficient frontier. However, a visual inspection of the stochastic cost verses value graph in Figure 6 indicates, the results are not nearly as black and white. In the stochastic model, when *true uncertainty* is accounted for, it illuminates the variety of possible outcomes for each alternative. When presented the traditional value model graph the cadets were inclined to select specific vehicle alternatives that seemed to obviously dominate the other alternatives. However, when presented with the stochastic value model graph they realized the impact that uncertainty had on the alternatives and became less interested in the alternative they were previously leaning towards. They noted that visually seeing the effects that uncertainty could have on the outcome of their decision made them more hesitant to go with the alternative that they had previously been most confident in choosing. This notion that the stochastic value model outputs are causing DMs to question their previous perceptions of alternatives is significant as it shows the DM's lack of complete information from the traditional model.

A one sample z-test of the means was executed to see if the results of the traditional model found in Figure 5 differed significantly from the stochastic model in Figure 6. The test compared the traditional model's point estimate result to the expected values of the stochastically assessed alternatives. The null hypothesis is that the there is no statistical difference between the results. This test used an alpha of 0.05 as this provided reasonable testing parameters of the z-statistic value from -1.96 to 1.96 (*Hypothesis Testing for Means & Proportions*. n.d.). The z-statistic value is solved using Equation 1 (Montgomery, D. C., & Runger, G. C., 2014).

$$Z_0 = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \tag{1}$$

In Equation 1, x-bar is the mean of the stochastic distribution, mu is the traditional representative value, sigma is the standard deviation of the stochastic distribution, and '*n*' is the sample size of five hundred. This equation was used to compute a z-statistic for each cost and value parameter. The test results indicated no significant difference between the results of the traditional and stochastic value models. These results coupled with the cadet-observed difference in dominance between the models shows that the data produced by the stochastic model does not need to be statistically different for it to provide decision-altering information to the DM. This aligns with the findings of Mittal et al. (2017) that a partially dominated solution could end up being a dominating solution. This suggests that the stochastic value model can provide the DMs with useful additional information in itself and that the complexity of the stochastic models is actually to the DM's advantage.

Following an assessment of the stochastic value model's utility, a Level 1 analysis was conducted to generate more valuable information. As a specific case study, the Toyota Corolla was compared to the Ford Escape. The Level 1 analysis supports the results of the stochastic value model, as it shows that the best decision between the alternatives is not intuitive. Table 1 contains the outputs of the Level 1 stochastic analysis between these alternatives.

Table 1: Level 1 Analysis Results for the Ford Escape to Toyota Corolla

Possible Outcomes	Number of Occurrences	Percent of Occurrence
Escape Dominates Corolla	15981	3.2%
Pareto Optimal (+)	406187	81.0%
Pareto Optimal (-)	7438	1.5%
Escape Dominated by Corolla	71895	14.3%

Table 2: Level 2 Analysis Results for Ford Escape to Toyota Corolla

Region	Occurrences	Percent Occurred
Zone 1	18	3.6%
Zone 2	243	48.5%
Zone 3	41	8.2%
Zone 4	77	15.4%
Zone 5	116	23.2%
Zone 6	6	1.2%

As shown in Table 1, the results of this comparison vary from the results of the traditional model. It shows that although eighty-one percent of the time the Escape and Corolla are pareto optimal, about fourteen percent of the time the Corolla dominated the Escape. These results provide evidence that due to *true uncertainty* the Corolla could have a lower cost as well as a higher value than the Escape. This non-zero possibility that the Corolla will dominate the Escape was a significant finding and showed the cadet that the Corolla had more potential than previously perceived.

A Level 2 analysis produced even more information, as seen in Table 2. These results display the comparison of the Escape's value and cost distributions to the Corolla's expected value and cost. This Level 2 analysis allowed the cadet to infer information they could not before, ultimately redefining their perceptions on the alternatives in question. This model shows that the Escape will produce a better than expected tradeoff an additional forty-nine percent of the time, as seen in zone 2. This showed that the cadet will be happy with choosing the Escape over the Corolla a total of fifty-three percent of the time (zones 1 and 2). Additionally, a worse than expected, but acceptable outcome is predicated to occur eight percent of the time (zone 3), while the cadet would be dissatisfied with the Escape fifteen percent of the time. This additional information enabled the cadet to redefine her perceptions with much more confidence. This information delayed the cadet's decision; however, they felt as if they were making a much more informed decision, increasing their clarity of action. This ability to increase the confidence of a DM is a significant finding as their confidence in the decision could be vital. This shows that before these techniques are applied a DM is making decisions based on incomplete information – information that could make them more comfortable with their decision. The stochastic value model alone is not enough either, they gained valuable information from the Level 1 and Level 2 analysis significantly improving their clarity of action.

In addition to these findings, this application highlighted a point of interest within the Level 2 analysis regarding the "comfortability trendline." Their response indicated a correlation between this trendline and the cadet's original budget. It was observed that the cadets with higher original budgets were more willing to go with an alternative that contained higher levels of uncertainty in the cost parameter. This led to the idea that this trendline is related to an aspect of opportunity cost that Richard Thaler refers to as the "endowment effect," where "people valued things that were already part of their endowment more highly than things… that were available but not yet owned," (Thaler, R., 2015). This was specifically observed in the fact that cadets were willing to sacrifice on cost to avoid losing the value but were less willing to pay more money in order to obtain an alternative with a higher value. This highlights a possible correlation of the DMs confidence to this endowment effect of opportunity cost.

3.2 Application to Squad-X DARPA Capstone

A SVM and Level 1 analysis were also applied to a USMA capstone project for DARPA called "Squad-X Micro-Breaching." The purpose of this capstone was to breach a small hole in a concrete wall as discretely as possible. These methods assisted the capstone team in deciding what tool should be used to create the initial breach. The traditional value model narrowed it down to a decision between a drill with a core-bit or a rotary hammer. Through SVM and Level 1 analysis they were able to decide. The results of the stochastic value model are displayed in Figure 7.

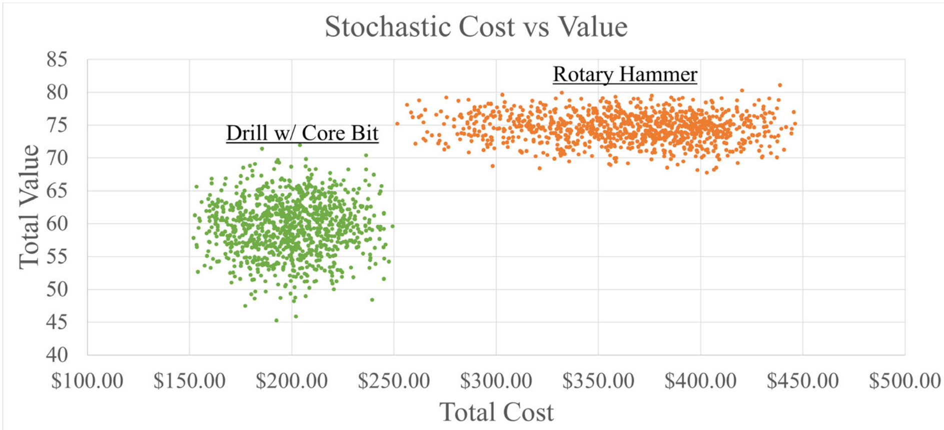


Figure 7: Stochastic "Squad-X Micro-Breaching" Alternative Value v. Cost

Through these decision-making techniques, they came to the decision that the rotary hammer was the best alternative. The SVM graph shows that the drill with a core-bit had much more uncertainty in its value parameters. In addition, the Level 1 analysis revealed "a 99.96 percent chance of the Rotary Hammer having a higher value while also having a higher cost," (Amalfitano, M., Shively, B., Finch, A., Sandlin, C., Wu, F., & Kelly, J., 2018). The team concluded that "Since total cost of the product is not as big of a focus for this point in the Squad X project, the obvious best breaching tool for the job is the Rotary Hammer," (Amalfitano, M., et al., 2018). Through Level 1 analysis the team was able to determine that the *true uncertainty* involved in their decision would not completely alter their decision. This shows that these approaches help real-life decisions and provide decision-makers improved clarity of action. Today, this decision has proven vital to the team's success. In fielding, the rotary hammer was a quick and effective solution, which leads credence to the applicability and utility of these decision-making techniques.

4. Results

The stochastic value model provides DMs with additional useful information indicating that the complexity of stochastic value models are beneficial to the DM. When asked about the usefulness of these models the cadets showed an interest in the traditional model when a decision was needed to be simplified and acted upon quickly. However, they specified that the stochastic models and follow-on analyses provided unmatched confidence when choosing an alternative due to the amount of data presented and representation of *true uncertainty.* Before these techniques are applied, DMs are facing uninformed decisions. With a SVM they gain enough information to change their perspective but lack complete confidence. Level 1 and Level 2 analyses provide an easy to understand and improved assessment of the alternatives, and ultimately increases the confidence and clarity of action for the DMs. Additionally, the original budget of the DM influences their comfortability in their decision, as the larger the original budget the more comfortable they are with uncertainties in cost. Finally, the alternative solutions that these decision-making techniques are providing are not only informative but have also shown to be in the DM's best interest.

5. Conclusions and Future Research

This study demonstrated improved clarity of action from using SVM and the Level 1 and 2 analyses. Further research is recommended into what this thesis coined the "comfortability trendline," it's correlation with opportunity cost and the endowment effect, and a more robust assessment of decision maker confidence. It is recommended that these two levels of analyses be applied to more real-world decisions to observe and quantitatively assess how this trendline and the DM's confidence are affected before and after the techniques are applied. This could shed more light on assessing this trendline and lead to a more in-depth and accurate level of analysis.

6. References

Amalfitano, M., Shively, B., Finch, A., Sandlin, C., Wu, F., & Kelly, J. (2018). *Breach Boys: Optimization of SQUAD-X Breaching Interim Report*. West Point, NY: United States Military Academy

Caddell, J., DuBois, P., Driscoll P., & Dabkowski, M. (2018). *Unmasking Uncertainty in Multi-Attribute Value Models (Level 1): Exploring Uncertainty on the Frontier* [PDF]. West Point, NY: Department of Systems Engineering United States Military Academy.

Caddell, J., DuBois, P., Driscoll P., & Dabkowski, M. (2018). *Unmasking Uncertainty in Multi-Attribute Value Models (Level 2): Illuminating the Trades we Expect* [PDF]. West Point, NY: Department of Systems Engineering United States Military Academy.

Howard, R. A., & Abbas, A. E. (2016). *Foundations of decision analysis*. Boston: Pearson.

How We Rank Used Cars. (n.d.). Retrieved from https://cars.usnews.com/methodology-used

Hypothesis Testing for Means & Proportions. (n.d.). Retrieved from http://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTest-Means-Proportions/BS704_HypothesisTest-Means-Proportions3.html

Lee M. D., & Dry M. J. (2006). *Decision making and confidence given uncertainty advice*. Irvine, CA: Cognitive Science Society, Inc.

MacCalman, A., & Parnell, G. S. (2016). *Multiobjective decision analysis with probability management for systems engineering trade-off analysis*. IEEE Computer Society.

Mittal, V., Lesinski, G., & MacCalman, A. (2017). *Selecting Robotic Power Solutions: A Case Study of Stochastic Value Modeling*. Herndon, VA: 6th Annual World Conference of the Society for Industrial and Systems Engineering.

Montgomery, D. C., & Runger, G. C. (2014). *Applied statistics and probability for engineers* (6th ed.). Singapore: Wiley.

Mousavi, S. M., Jolai F., Tavakkoli-Moghaddam, R. (2011). *A fuzzy stochastic multi-attribute group decision-making approach for selection problems*. Springer Science.

Packard, M. D., Clark, B. B, & Klein, P. G. (2017). *Uncertainty types and transitions in the entrepreneurial Process.* INFORMS.

Parnell, G. S., Driscoll, P. J., & Henderson, D. L. (2011). *Decision making in systems engineering and management* (2nd ed.). Hoboken, NJ: Wiley.

Savage, S. L., & Danziger, J. (2012). *The flaw of averages: Why we underestimate risk in the face of uncertainty*. Hoboken (New Jersey): Wiley.

SIPmath. (n.d.). Retrieved from https://www.probabilitymanagement.org/sipmath

Thaler, R. H. (2016). *Misbehaving: The making of behavioral economics*. New York: W.W. Norton & Company.