

## Simplified Simulation of Torsional Stiffness in Side Rail-Cross Member Bolted Joints

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**Abstract:** It proposes and implements a simplified modeling procedure of the torsional stiffness of side rail-cross member bolted joints used in motor vehicles, which would allow simulating the mechanical behavior of the chassis of vehicles with simplified models, having short-term solutions and post-processing of results without losing accuracy and detail thereof. The particularities of the torsional stiffness of these joints have been little considered, despite knowing its influence on the mechanical behavior of the chassis and performance. The proposed modellings are implemented through the FEM.

**Keywords:** Torsional Stiffness, Chassis Modeling FEA, Bolted Joint

### 1. Introduction

For decades, studies and modeling have been performed on the mechanical behavior of bolts, screws and their use in structural joints, emphasizing bolts and threads basically (Bickford, 2018; Fukuoka, 1994; Norton, 1998; Matienzo and Pereiro, 2006). Some of the most recent works published in the last few years are found in Ref. (Grant and Hornish, 2010; Maggi et al., 2005; Nasraoui et al., 2012; Odyjas and Karliński, 2018; Abid et al., 2016). Another group of studies usually gather models and simulations of various bolted joints (Feng et al., 2004; Kim et al., 2007; Korolija, 2012; Razavi et al., 2007) using several methods, with highly detailed and accurate results of all elements of the joint, valid for general design.

The torsional stiffness incidence of the side rail chassis on the behavior and performance of a motor vehicle is poorly studied and supported at present. In (Fukuoka, 1994; Grant and Hornish, 2010; Nasraoui et al., 2012) the incidence of torsional stiffness of the chassis in the behavior of the vehicle is studied, and although no definitive conclusions are reached, they pave the way to continue deepening into this important and yet little studied topic. More recently Biradar et al. (2008) a detailed work was carried out on the optimization of the space frame chassis, but did not consider the critical role of torsional flexibility of the frame in its design and optimization.

The torsional behavior of bolted joints is actually well-studied and well-founded. Everything indicates that it was F. Romanow (1988) who first published detailed studies on this topic, applied to joints of chassis members, even

analyzing their stress behavior. But ever since, new and varied types of bolted joints have emerged (and are still emerging).

Also, mathematical modeling is currently widely used in designs, calculations and studies of vehicle chassis, using often very complex models, consisting of hundreds of thousands and millions of equations to achieve accuracy and most details of the systems analyzed (Genta and Morello, 2009; Genta and Morello, 2009; Odyjas and Karliński, 2018; Shigley et al., 2004). Difficulties often arise in the use and treatment of these large models, especially with the common use of modern microcomputers, PCs.

This work presents and supports a modeling by FEM of these joints, contributing to decrease hundreds of thousands degrees of freedom involved (DOF); thereby saving time and increasing the simplicity of the models, without losing accuracy or details of the results. They use modern BEAM type dimensional finite elements recently developed. The main aspect that affects the construction of these simplified models is considering therein torsional stiffness of bolted joints present in the chassis, an aspect which is given most attention in this work.

## 2. Materials and Methods

### 2.1 Theoretical foundation

In the traditional theory of the torsion of open thin-walled bars, the case of so-called Limited torque has been defined (Figure 1), wherein one end of the bar is completely supported and the other free, applying load on it. The torsional moment of inertia of the profile, considering the effect of bimoment B and the profile buckling phenomenon is given by,

$$I_n = (1 + \alpha) I_o \quad (1)$$

Where:

$\alpha$ : Coefficient that considers the effect of the bimoment and the emergence of additional stresses  $\sigma_\omega$  it produces, for a bar simply supported under the action of a torque at its free end. That is to say, with limited torque. (Fig. 1)

$$\alpha = \frac{\tanh(k \cdot L)}{k \cdot L - \tanh(k \cdot L)} \quad (2)$$

L is profile length and k is flexion-torque coefficient.

$$k = \sqrt{\frac{G \cdot I_\omega}{E \cdot I_o}} \quad (3)$$

$I_\omega$  is sectorial moment of inertia of the profile and  $I_o$  is torsional moment of inertia of the open thin profile in free torque.

$$I_o = \alpha' \cdot \frac{1}{3} \sum_{i=1}^n (t_i \cdot s_i^3) \quad (4)$$

$\alpha'$  is an coefficient that considers the effect of the corners of the profile in its moment of torsional inertia, being  $\alpha' = 1.25$  [Beam I] y  $\alpha' = 1.12$  [Duct Beam];  $n$  is number of straight rectangular sectors ( $t$  xs) in which the cross section can be divided.

The torque angle  $\theta$  of free end is given by,

$$\theta_{\max} = \frac{M_t \cdot L}{G \cdot I_n} \quad (5)$$

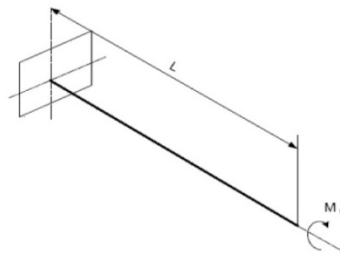


Figure 1. Schematic of a bar or a beam under limited torque.

Especially when the beam support is perfect, i.e. ideal. For a bolted joint the flexural rigidity can be considered to be approximately perfect (Genta and Morello, 2009; Norton, 1998) but torsional stiffness can vary over a wide range, depending on the type of joint used.

In determining the torsional stiffness of a bolted joint, as significant parameter (Norton, 1998) torsional stiffness coefficient  $[\delta]$  is defined for it, given by:

$$\delta = \frac{1}{\alpha} \left( \frac{M_t \cdot L}{G \cdot I_n \cdot \theta} - 1 \right) \tag{6}$$

This coefficient measures the degree of distance from the torsional stiffness of the joint on an ideal support. The new torsional moment of inertia of the bar with a specific bolted joint is now given by,

$$I_{nr} = (1 + \alpha\delta) I_o \tag{7}$$

So that the angle  $\theta$  of the free end is given by,

$$\theta_{\max} = \frac{M_t \cdot L}{G \cdot I_{nr}} \tag{8}$$

Thus, for  $\delta = 0$ ,  $I_{nr} = I_o$  it is free end torque, i.e. without any limitation of the ends to gyration and twist of profile sections. In fact, it implies that there is not joint at the ends.

For  $\delta = 1$ ,  $I_{nr} = I_n$  it is torque with a completely supported end. The other end is considered free and loaded with the torque  $M_t$ . This is the ideal limited torque.

## 2.2 Modeling of joints.

Figure 2 shows two of the joints studied by Romanow (1988) in their torsional behavior, characterized by the shear modulus of torque  $\delta$ .

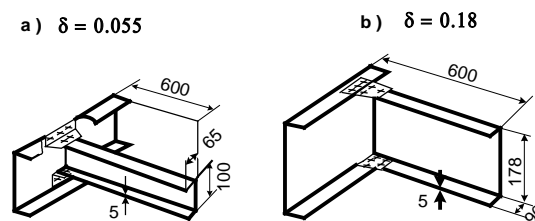


Figure 2. Two of joints studied in Romanow (1988), all distance in [mm].

In figure 3 a model of Finite Elements of the joint of Fig. 2 a) is proposed, with the use of the modern BEAM type one-dimensional elements (Fig. 6), with which the model is greatly simplified compared to the use of SOLID or SHELL type traditional elements.

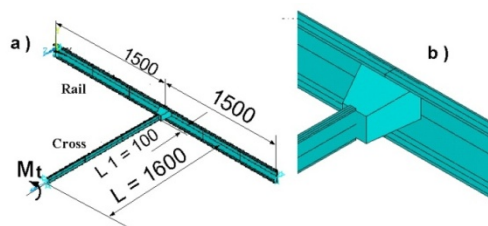


Figure 3. a) Model of FEA with joint stiffness  $\delta = 0.055$ . b) Details of the joint.  $M_t = 100 \times 10^3 \text{ N}\cdot\text{mm}$ .

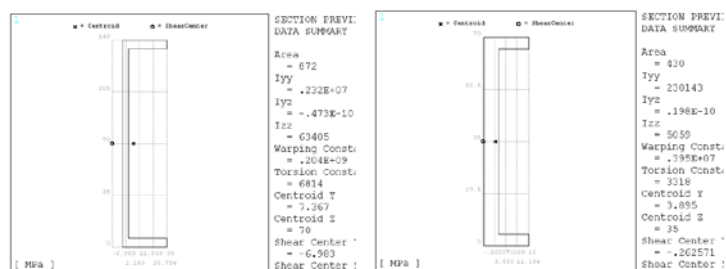


Figure 4. Geometric properties of: a) rail 140 x 30 x 6 mm and b) cross member 70 x 15 x 5 mm.

Some of the main characteristics of the BEAM unidirectional elements used are:

- Isoparametric with cubic interpolation functions (Hermitian).
- 25 degrees of freedom (DOF).
- Consideration of the warping of the cross sections of the beams. Vlasov equations for open thin-walled profiles.
- And Timoshenko's Theory for beam calculation.
- Calculation of bimoment and delineating profiles.
- Possibility of relaxation of different DOFs.

To take into account different torsion stiffnesses of bolted joints simulated with these finite elements, the ability to vary their torsion stiffness must be incorporated to them, in correspondence with the actual torsional stiffness of the joint. Therefore, it is proposed to model the joint through the combination of two unidirectional finite elements:

- BEAM element (Fig. 6a).
- Linear SPRING torque element (Fig. 6b).

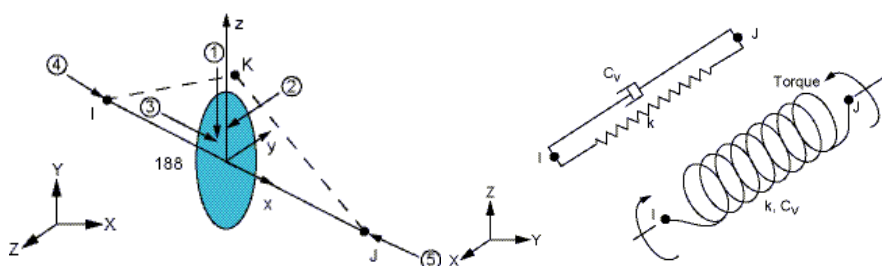


Figure 6. unidirectional finite elements used: a) one-dimensional BEAM element and b) Linear SPRING torque element.

This appears in the area of Fig. 3 as a truncated pyramid. BEAM element is released from its torsion stiffness, which is assumed by the torsion spring constant  $K$ , a built-in SPRING element. The relationship between this  $K$  and coefficient  $\delta$  characteristic of the joint is given by,

$$K_{spring} = GI_o \left( \frac{1 + (1 + \delta + \alpha\delta)}{\alpha(1 - \delta)} \right) \quad (9)$$

Where, for the cross member created it is obtained,

$$K_{spring} = 2065240 \times 10^6 \frac{N \cdot mm}{rad}$$

The model proposed in Fig. 3 has only 8 finite elements, 16 nodes and 356 degrees of freedom DOF. While a similar model built with SOLID type volumetric elements, would require about 80 000 elements with hundreds of thousands of degrees of freedom DOF. Being able to notice the vast simplification resulting from the use of this kind of model proposed herein.

Using theoretical equations provided by Romanov (1988), where the torsion angle is calculated  $\theta_{theoretical}$  of the free end of the cross member, obtaining

For  $\delta=0.055$  and  $M_t=100 \times 10^3$  N-mm.

$$\theta_{max} = \frac{M_t \cdot L}{G \cdot I_{nr}} = \frac{M_t \cdot L}{G \cdot (1 + \alpha\delta) I_o}$$

$$\theta_{theoretical} = 0.6233rad = 34.5^\circ$$

Resolving the finite element model of the modeled joint (Fig. 3), the results shown in Fig. 8a and Fig. 8b are obtained. From which,

$$\theta = 0.5877rad = 33.67^\circ (\delta = 0.055)$$

$$\sigma_{equiv}^{max} = 275MPa$$

$\sigma_{equiv}^{max}$  is the equivalent stress according to the Huber–Misses' resistance criterion of the part of the cross-member outside the joint. The similarity of the angles of twist  $\theta$  of the free end of the cross member, between the theoretical and the one calculated by the modeling performed can be observed. The relative difference between the two is,

$$e = 2.4\%$$

Another comparison of interest is between the angles  $\theta$  calculated by the previous modeling, compared to a model without SPRING element and only with the BEAM. So a complete building-in between nodes of the BEAM elements of the cross member and rail would be considered. The results of this last simulation are,

$$\theta = 0.60634rad = 34.74^\circ (\delta = 0.055), \text{ (beam-beam)}$$

$$\sigma_{equiv}^{max} = 415.34MPa$$

The similarity between the torsion angles obtained, while there is a great difference in  $\sigma_{equiv}^{max}$  is in the cross member. This second modeling without the spring element in the joint represents the joint of the BEAM elements and the cross member as a perfect built-in through its nodes, i.e. a traditional beam-beam joint.

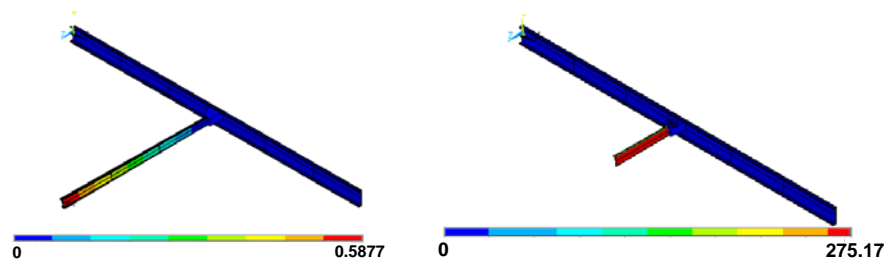


Figure 8. Result with  $\delta = 0.055$  for a) angular displacements (ROT Z) [rad] and b) equivalent stresses [MPa].

Similarly, the following joint model corresponding to that shown in Fig. 2 b) was constructed with  $\delta=0.18$ , model shown in Fig 9. The results of interest are the following.

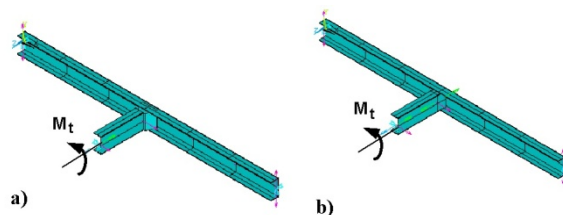


Figure 9. Model of a joint with  $\delta=0.18$ . a) Joint with BEAM and SPRING elements. b) Joint with BEAM element.

For  $\delta = 0.18$  and  $M_t=100 \times 10^3$  N-mm.

$$\theta_{\max} = \frac{M_t \cdot L}{G \cdot I_{nr}} = \frac{M_t \cdot L}{G \cdot (1 + \alpha\delta) I_o}$$

$$\theta_{\text{theoretical}} = 0.045 \text{ rad} = 2.58^\circ$$

Resolving the finite element model of the modeled joint with presence of the suitable SPRING element (Fig. 10a), the following results were obtained.

$$\theta = 0.133 \text{ rad} = 7.62^\circ (\delta = 0.18)$$

$$\sigma_{\text{equiv}}^{\max} = 69.67 \text{ MPa}$$

Comparing now the angles  $\theta$  calculated by the above modeling, as compared to a model without SPRING element and only with the BEAM (figure 9\_b), there is,

$$\theta = 0.0752 \text{ rad} = 4.30^\circ, \text{ (beam-beam)}$$

$$\sigma_{\text{equiv}}^{\max} = 54.08 \text{ MPa}$$

Observing different results with the use of each model, more difficult to generalize.

### 3. Analysis of results

The following table summarizes the main results obtained in this work. The first thing to note is that in modeling, consideration of the torsional stiffness of the joints through the coefficient  $\delta$ , is only significant for joints with high  $\delta$  values, such as  $\delta = 0.18$  and higher. In these cases, the difference of  $\theta$  not considering  $\delta$  but the beam-beam joint between BEAM elements is 43 %. In these cases, then modeling with SPRING elements incorporated into the joint is necessary. While for joints with low  $\delta$  (like  $\delta = 0.055$ ), may obviate the placement of this element and work with the beam-beam joint.

Due to maximum equivalent stress values in the cross member, the behavior of the models is exactly the opposite. For high  $\delta$  the use of the SPRING element can be ignored in the joint; while for low  $\delta$  considering it is essential. As inferred from stress values for each case provided in Table 1.

Regarding the comparison of the angles  $\theta$  obtained by modeling, regarding  $\theta_{\text{theoretical}}$  achieved with the use suitable  $\delta$  in the models; always good agreement with the corresponding  $\theta_{\text{theoretical}}$  is obtained. While the model without the element of the SPRING type, with high  $\delta$  values ( $\delta = 0.18$ ), great differences occur regarding the corresponding  $\theta_{\text{theoretical}}$ .

For example, for a joint with  $\delta=0.18$ , model without SPRING (beam-beam):  
 $\theta = 0.0752=4.30^{\circ}$  ( $\theta_{\text{theoretical}} = 0.127 \text{ rad} = 7.30^{\circ}$ )  
 Difference:  $e = 69 \%$ .

Table 1. Main results obtained in this work.

<b>L=1600 mm</b>				
Type of joint	$\theta$ Modeling	$\theta$ Theoretical	Error % between	Stress** MPa
With $\delta=0.055$	0.5877 rad $33.67^{\circ}$	0.6022 rad $34.50^{\circ}$	2.4	<b>275</b>
Built-in (beam-beam)	0.60634 rad $34.74^{\circ}$	0.594 rad $30.03^{\circ}$	3.06*	<b>415.34</b>
With $\delta=0.18$	0.133 rad $7.62^{\circ}$	0.127 rad $7.30^{\circ}$	4.5	<b>69.57</b>
Built-in (beam-beam)	0.0752 rad $4.30^{\circ}$	0.0983 rad $5.63^{\circ}$	43*	<b>54.08</b>

\*between modelsof Finite Element (FE)  
 \*\* Modeling in cross member

All above refers to a total length of the cross member of  $L = 1600$  mm. But it is of interest to know the behavior of the joints for different L lengths of the cross member. This study is conducted and Fig. 10 to Fig. 11 show some of the main results obtained.

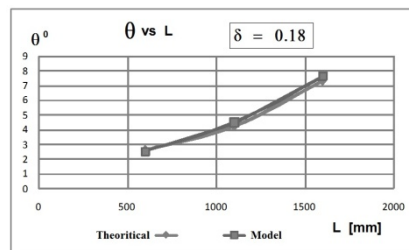


Figure 10. Graph of  $\theta$  vs. L.  $\delta = 0.055$  (Romanow). Theoretical vs. Model. Error: 8.6-13.04 %.

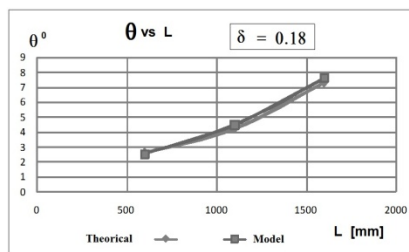


Figure 11. Graph of  $\theta$  vs. L.  $\delta = 0.18$  (Romanow). Theoretical vs. Model. Error: 1.55 %.

In summary, it can be observed that regard in the models of spring-like elements in the joint, with its constant K in correspondence with its characteristic torsional stiffness  $\delta$ , is a must in all cases analyzed. In some cases, for proper calculation of the angle of twist  $\theta$ ; and to calculate the maximum stresses in the cross member, in other cases.

It can be seen that the differences between the angle  $\theta$  the free end of the cross member, calculated by modeling considering the K of the SPRING suitable, compared to  $\theta_{\text{theoretical}}$ , are eligible for ranges  $\delta$  and L studied. These differences are:

$$e = 1.55-13.04 \%$$

This shows the adequacy of the cross member-rail bolted joints modeling proposed here to simulate torsional behavior properly. With which significant simplifications in the construction and development of models of chassis and frames can be achieved, with consequent saving of efforts, machine time and post-processing of results.

#### 4. Conclusions

1. The torsional stiffness of the joint with  $\delta=0.055$  is 6 times more flexible than the joint with  $\delta=0.18$  (Fig. 2). Therefore, there is a big difference in torsional stiffness between both types of joints.
2. Stresses were calculated only through modeling by the FEM, resulting in great value differences when working with the  $\delta$  analyzed (0.055-0.18), as compared to  $\delta=1$  (limited torsion).
3. For the calculation of the angles of twist  $\theta$  of the free end of the cross member, with bolted joints modeling, the use of the SPRING type finite element is required only for joints with high values  $\delta$ .
4. But for modeling equivalent stresses of the beams involved, the opposite occurs: it is essential to use the SPRING element for joints with low  $\delta$ .
5. Therefore, for proper modeling the cross member - rail bolted joints used in motor vehicles, with the use of one-dimensional finite elements of BEAM type, it will always be necessary the addition at the joint of a torsional SPRING-like element. With its constant K corresponding to the coefficient  $\delta$  of the joint.
6. The relationship between the spring constant K of the SPRING element and the torsional stiffness coefficient of the joint is provided  $\delta$ ; so as to develop a modeling of the joint properly.
7. Modeling of bolted joints using BEAM-like and SPRING-like one-dimensional finite elements, allows simulating the behavior of the joint and structure properly, considering the actual effect of torsional stiffness characteristic of the joints considered. With great savings and simplification of the models, compared to the use of other types of finite elements.

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