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Solving Capacitated Vehicle Routing Problem Using Two Phase Heuristic Method

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Abstract: This paper proposes a modified approach of sweeping to solve Capacitated Vehicle Routing Problem (CVRP). CVRP is a combinatorial optimization problem that has an additional constraint to the classic Vehicle Routing Problem (VRP) of limited capacity of the trucks. A set of trucks deliver goods to a given set of customers in which the total travelled distance of the trucks is minimized while the customers demand is fulfilled, and the truck capacity is not exceeded. Even though the problem itself may be complex to solve directly, it can be split into several Traveling Salesman Problems (TSP) using an approach referred to as sweeping, which segments the delivery zones in angular sectors, each corresponding to the delivery capacity of a vehicle. In this paper, a new form of sweeping that allocates vehicles to both angular and radial clusters of locations is proposed to improve solutions to problem instances with location clusters across the radial dimensions in terms of the total distance travelled. The experimental results indicate that the proposed approach reduces traveling distance by 2-4% for instances with clusters on radial positions, while indicating that it increases traveling distance in cases with no clusters present.

Keywords: Capacitated Vehicle Routing Problem, Vehicle Routing Problem, Sweeping, Travelling Salesman Problem

1. Introduction

The rise in online ordering and the high transportation costs associated with the delivery of these orders urges the need of route optimization for vehicles delivering these goods. An optimized route for the delivery can decrease the distance travelled by the vehicles significantly and therefore decrease the cost associated with the delivery. The problem of finding an optimal route for a set of vehicles visiting a set of locations is referred to as the Vehicle Routing Problem (VRP). A more general form of this problem is the Capacitated Vehicle Routing Problem (CVRP) where vehicles have limited capacities, locations have demands, and solution routes must guarantee the satisfaction of all demands while not exceeding vehicle capacities (Dantzig and Ramser, 1959). Applications to CVRP include the delivery of goods to customers using courier services such as FedEx home delivery service and UPS delivery.

Several approaches have been proposed over the years to solve CVRP. Most of these approaches are heuristic approaches since exact algorithms such as branch and bound and branch and cut require non-polynomial time to solve large size problems. However, heuristic approach does not explore the entire search space but rather tries to find an optimal solution based on the available information of the problem.

One of the main difficulties with CVRP is performing routing subject to vehicle capacity constraints. These two concerns can be separated using a two-phase heuristic approach known as the Sweeping Algorithm (SA) (Suthikarnnarunai, 2008) proposed by Gillett and Miller (1974). This algorithm is constructed to generate routes for vehicles in which a solution to Travelling Salesman Problem (TSP) takes place in the second stage. In this approach, an initial location is chosen (conventionally the depot) and locations are sorted according to their angular position to this location. A "sweep" is then started from angle 0 in the clockwise direction, including as many locations as possible until the capacity of a certain vehicle is exhausted. This vehicle is then assigned the just swept sector of locations. The process is then repeated, this time starting from the angular position where the previous iteration ended, to determine the delivery sector for the next vehicle and so on. SA is a widely used method for clustering that basically clusters the nodes solely by polar angle, this can form a problem if the nodes are widely separated but have less angular distance as they can be assigned in the same cluster, and this reduces the optimality of the resulting cluster.

In this paper, CVRP problem instances with clusters on radial positions are considered. Such instances tend to be common in practice. For example, many cities around the world contain residential blocks separated by rings of roads, including London (3 ring roads), Houston in Texas (3 ring roads) and Beijing (6 ring roads). In such cities, allocating vehicles per rings will be more optimal as it will avoid repetitive crossing of ring roads, which may increase distance significantly. SA does not take this into account. A modification to SA is proposed that can improve the handling of problem instances with clusters on radial positions. Specifically, a conventional sweep round is performed to split locations into angular sectors. And then allocating each sector to a group of vehicles (as opposed to a single vehicle in SA), and performing a second round of sweeping, this time radially, to divide the sector into radial regions to be allocated to vehicles. This is proposed to give minimal travelled distance for vehicles and better clustering of locations**.**

The remainder of this paper is organized as follows: Section 2 provides a literature review on the CVRP. The research methodology for solving CVRP problem is illustrated in Section 3. Experimental settings and results are presented in Section 5. Research conclusion and future work directions are provided in Section 6.

2. Literature Review

CVRP is a well-known integer programming problem which is classified as an NP-hard problem. It is concerned with finding an optimized route for delivery for a set of vehicles leaving from the depot. All vehicles must start and return to the depot while each location must be visited exactly once by one vehicle only, while the total distance (or an alternative cost function) is minimized. The numerous applications of CVRP in industry and management of distribution systems such as the delivery of goods to customers have led to the implementation of various solutions over the years. Solutions tend to fall into one of two groups; exact and heuristic (approximate) methods. Exact methods are proved to give optimal solutions but are developed only to solve small scale VRPs because of their high computational time. Most of these methods are based on the branch and bound approach which computes every possible solution and determines the most optimal. Several exact procedures for solving VRPs including branch and bound are reviewed in (Laporte, 1992).

Heuristic methods however do not guarantee optimality but nevertheless tend to produce good solutions. One simple heuristic method is the saving algorithm (Clarke and Wright, 1964) which is based on savings distance that is obtained by joining two routes into one route. Heuristic methods can be classified into three categories: (1) construction heuristics which build a feasible solution while minimizing solution cost, (2) improvement heuristics which perform vertex exchanges within or between vehicle routes to improve the current solution and (3) a two-phase heuristic which breaks the problem into two tractable sub-problems and joins their solutions through route reconstruction.

Two phase heuristics can be of two types: (1) cluster-then-route and (2) route-then-cluster methods. In the first case, vertices are first organized into feasible clusters, and a vehicle route is constructed for each of them, while in the second a tour is first built on all vertices and it is subsequently segmented into feasible vehicle routes (Lal et al., 2009). (Boyzer et al., 2014) proposed a heuristic algorithm for the VRP based on cluster first-route second. They used fuzzy C-means clustering followed by tabu search to improve generated solutions. An example of the cluster first-route second is SA that clusters the locations based on their polar angle. (Nurcahyo et al., 2002) investigated the capability of SA in generating routes for public transport. Some extensions to this algorithm were proposed, including using the nearest neighbor and various reference points to improve the results (Akhand et al., 2007). Metaheuristic paradigms have extended simple local search approaches by devising techniques that allow for escaping from local minima and continue the search towards possible solutions (Toth and Vigo, 2002). These approaches produce better results but take more time. Some of these approaches are used in solving CVRP such as genetic algorithm that have been shown to be capable of solving VRPs (Baker and Ayechew, 2003). Simulated annealing is also used to solve the VRP (Hiquebran et al., 1993), and tabu search which has been widely used throughout the literature (Braysy and Gendreau, 2002).

3. Methodology

3.1 CVRP using Sweeping Approach

The classical SA is a well-known clustering method that clusters locations according to their angular position so that locations which are close together can be served by the same vehicle. SA is performed using the following steps:

Sweeping Approach (SA)	
Step 1	Start from the depot and locate it as reference (0,0)
Step 2	Compute the coordinates for each location depending on the depot
Step 3	Start sweeping from the depot by increasing the polar angle
Step 4	Assign locations to vehicle
Step 5	Stop when capacity of vehicle is exceeded
Step 6	Start another sweep from the last location visited by the vehicle and assign locations to new vehicle
Step 7	Continue until all locations are assigned to vehicles

This breaks down CVRP into multiple TSP where each vehicle goes through a specified route and returns to the depot. Each route can then be minimized using classical TSP. Limitations of SA is that it only groups the nodes based on their polar angle and assigns a vehicle for each group. This grouping may be sub-optimal because it assumes that nodes with similar polar angles are close, while in reality they can be arbitrarily far apart. The proposed methodology to solve this problem is a modified SA, it considers far away points and problem instances with clusters on radial positions for better clustering of nodes to clusters. There exist many real-world examples to this problem such as natural barriers of rivers and mountains.

3.2 Modified Sweeping Approach

The modified sweeping is performed in two phases where the first phase is assigning vehicles to locations using modified SA which is applied as follow:

Modified Sweeping Approach (Modified SA)	
Step 1	Perform classical SA
Step 2	Try different combinations of vehicles where each combination consists of merging two vehicles $C_i = \{v1 + v2, v3, v4, .., n\}$
Step 3	Take one combination and assign the merging vehicles into a super vehicle $SV = \{v1 + v2\}$
Step 4	For this super vehicle start a sweep but this time radially.
Step 5	Assign locations to first vehicle
Step 6	Stop when capacity of first vehicle is exceeded
Step 7	Start another radial sweep from the last location visited by the first vehicle
Step 8	Assign locations to the second vehicle
Step 9	Stop when capacity of the second vehicle is exceeded
Step 10	Repeat steps 3-9 by taking different combinations of vehicles

The second phase is route optimization using TSP, each vehicle will now visit specified locations and CVRP will break down into multiple TSPs which is easier to solve. An off-the-shelf TSP optimizer is then used to optimize the route of each vehicle. Figure 1 shows a comparison between SA and modified SA.

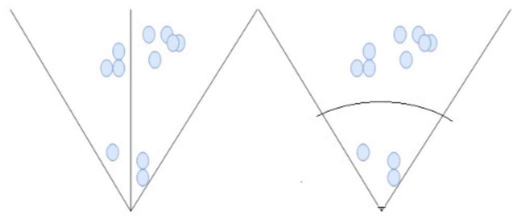

Figure 1. Comparison between SA and Modified SA

4. Experimental Results

A numerical experiment is conducted to evaluate the performance of the proposed approach. 40 different locations are selected randomly. Each location represents a customer that has a demand of 1 package and can be served by only one of 5 vehicles. Each vehicle has a capacity of 8 and must leave from the origin point and return to this point. Figure 2.a shows the assignment of trucks to locations when using SA resulting in 5 sectors, each containing a set of locations that are to be visited by a vehicle. A TSP is then performed for each sector to develop a near-optimal route for the vehicles resulting in an overall minimized travelled distance.

Figure 2.b shows the same 40 locations and the same 5 vehicles leaving the origin point with same capacity of vehicles and same demand of locations. Classical SA is performed, and then different combinations of vehicles are taken resulting in the merging of vehicles at sectors 3 and 4 into Super Vehicle (SV). It can be shown from the graph that these two sectors have clusters on radial positions which may indicate why these sets of vehicles were chosen to be merged. A radial sweep is then performed to this SV to break it down into two vehicles, locations are then assigned to the first vehicle until the capacity of this vehicle is exceeded. Another radial sweep is done starting from the last location visited by the first vehicle, and assigning locations to the second vehicle until the capacity of the second vehicle is exceeded. A TSP is then performed just like in SA except that the merged sectors would have different routes of vehicles now.

The results of the total distance travelled in SA is 16.13, while in the modified SA it is 15.53. An overall distance saving of around 4% when performing modified SA.

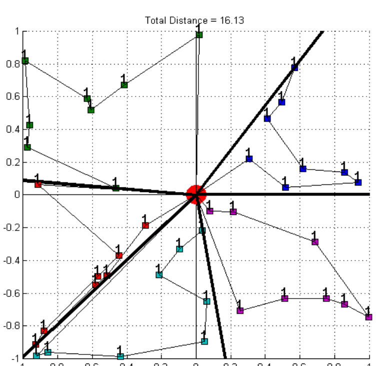

Figure 1.a Performing SA

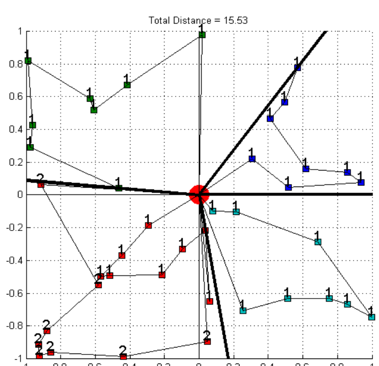

Figure 2.b Performing Modified SA

9 different instances were taken to test for the performance of modified SA with respect to total distance travelled. The instances taken varied between instances containing clusters on radial positions and instances with no apparent clusters. Table 1 summarizes the results of total distance travelled for problem instances with strong radial clustering by using both classical SA and modified SA. The proposed method achieved 2-4% saving in total distance travelled on average for the

targeted subclass of problems. Note that the instances taken are of small size resulting in this decrease in distance, by taking larger instances it is expected that there will be higher improvement of total distance travelled.

On the other hand, Table 2 shows the results of total distance travelled for problem instances with poor radial clustering. These instances had no apparent radial clustering and the locations were randomly placed. When performing modified SA, the results showed an increased travelled distance which indicates that modified SA did not work in this case and it did not give any improvements.

An example of both cases is shown in Figure 3 and 4. Figure 3 shows an instance with locations on radial positions, where sectors 4 and 5 were combined in one SV resulting in a decreased distance. These two sectors were combined because in these sectors there are two apparent clusters, one near the depot and one far from the depot, so performing modified SA would give an improvement in this case. While Figure 4 shows instance with no apparent clusters of locations, modified SA was performed for this instance and it resulted in increased distances indicating that this algorithm did not work for such instances.

Table 1. Representative of instances with strong radial clustering

Distance using SA	Distance using modified SA	Improvement
16.13	15.53	3.9%
15.29	14.90	2.6%
14.51	14.17	2.4%
16.11	15.75	2.3%
15.80	15.44	2.3%

Table 2. Representative of instances with poor radial clustering

Distance using SA	Distance using modified SA	Improvement
15.43	17.07	-10.6%
14.69	16.01	-9.0%
15.09	15.97	-5.8%
14.17	14.66	-3.4%

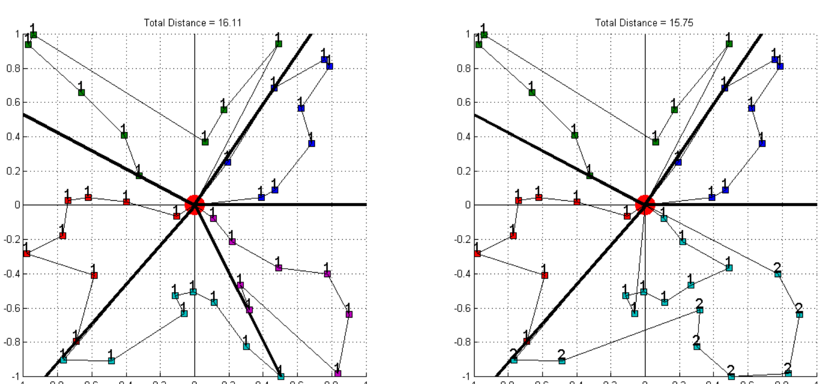

Figure 2. Performing SA and Modified SA for Instances with Clusters

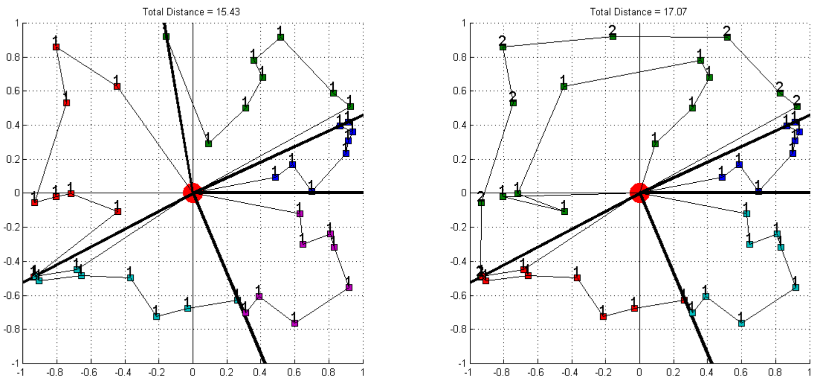

Figure 3. Performing SA and Modified SA for Instances without Clusters

5. Conclusion and Future Work

Different heuristics and meta-heuristics have been proposed to solve CVRP within a reasonable computing time. One of these heuristics is the sweeping algorithm that clusters locations based on their geographical distance. However, it only considers the angular position of locations, but gives good results generally. This algorithm does not consider far away points when performing sweeping especially when there are clusters on radial positions. A modified sweeping algorithm was proposed that accounts for such instances, where classical sweeping is performed followed by radial sweeping. The results showed minimized travelled distance of about 2-4% when using this approach compared to the classical sweeping but only for instances where there are clusters on radial positions. Small instances are taken resulting in these percentages. It is expected that when taking larger instances these percentages would increase and the improvement would be higher. Future work includes applying this approach for larger instances where different demands of locations are taken, and different capacities of vehicles are considered. The performance of the modified SA is only evaluated using the total distance travelled. Other metrics such as the computational time it takes and the selection of the combination of vehicles to choose will be used to provide comprehensive evaluation for the modifies SA.

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