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SEIR Models for Food Contamination Events

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Author Note: Jessye Talley has research expertise in stochastic and deterministic modeling of supply chains using stochastic programming, Markov chains, differential equations, linear programming, and queueing theory. She has a strong interest in the safety and defense of food supply chains which she modeled using compartmental models to show the progression of the illness of a consumer from eating a contaminated food product and the effects of interventions. Her current research interest consists of applications in humanitarian relief, emergency preparedness and response to address ports, healthcare, and food supply chain safety and defense.

Abstract: Food contamination outbreaks are becoming more prevalent in today's society. These outbreaks can occur due to contamination at any point during the food supply chain. This in turn can lead to many illnesses that cause a loss of confidence in the food chain. Two main objectives are considered for this research. The first objective is to consider the food products that are linked to illness. The second objective focuses on determining the optimal food removal policy. Two deterministic differential equations models are used to model these processes. As a result, we are able to see the progression of food through various stages and see how shelf life plays a role in the number of consumers that become ill. We are also able to see the optimal time to remove products from the shelf based on different food types to lessen the number of consumers that will become ill.

Keywords: Food, Contamination, Illness

1. Introduction

From 2009 to 2010, there were 1,527 food outbreaks reported by the United States (U.S) public health departments, which contributed to 7.8% of deaths (Centers for Disease Control and Prevention, 2014). Since the September 11th terrorist attacks, there has been an increased concern about the vulnerability associated with the food supply chain. Implementation of intervention strategies, such as recalls, can help in preventing the distribution of food with impurities because of a vulnerable system. This concern as well as recurring instances of illness has made food safety and defense a top priority in this country. Although previous research has shown developments in understanding food contamination events, many use only one solution approach such as simulations or growth and probabilistic models we consider using compartmental models.

2. Methodology

2.1 Case 1: Deterministic Differential Equation model with non-constant population, purchasing and consumption behavior (Food)

2.1.1 Model Description and Assumption

A deterministic ordinary differential equation progression model is developed to illustrate how contaminated food moves through the food supply chain. The model links illness in consumers to food products sent to a distribution channel. The phase of progression is as follows: (1)The total amount of contaminated food products is denoted by (A); (2) The number of food products purchased from a distribution channel that can lead to consumer illness is denoted by (P); (3) The number of contaminated food products consumed is denoted by (C); (4) The number of food products that cause illness is denoted by (I). This model is different from the original SEIR model because it uses food instead of people as the population. The population of food is finite and known. Based on the flow of each stage, we construct a system of differential equations (Figure 1).

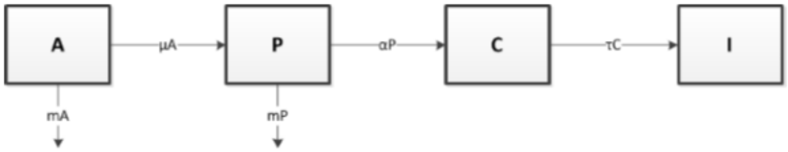

Figure 1. SEIR Model Flow Deterministic Differential Equation

Table 1 presents the parameters used for the Case 1 model

Table 1. Case 1 Model Notation

Symbol	Description
$A(t)$	The number of contaminated food products are available on shelf at time t
$P(t)$	The number of food products that are purchased from Distribution Channel (DC) at time t.
$C(t)$	The number of contaminated food products that are consumed at time t.
$I(t)$	The number of contaminated food product consumed that are linked to illness at time t.
m	The shelf life of the product.
μ	The rate which food products are purchased.
α	The rate which food is consumed.
τ	The rate at which a contaminated food product is linked to illness.
t	Number of days in a food outbreak.

2.1.2 Model Formulation

$$\frac{dA}{dt} = -mA(t) - \mu A(t) \tag{1}$$

$$\frac{dP}{dt} = \mu A(t) - mP(t) - \alpha P(t) \tag{2}$$

$$\frac{dC}{dt} = \alpha P(t) - \tau C(t) \tag{3}$$

$$\frac{dI}{dt} = \tau C(t) \tag{4}$$

A finite set of food items are available that exit the compartment by a purchase with rate μ or reaching shelf life with rate m (Equation 1). Once food reaches the purchase compartment, products can reach their shelf life with rate m or through consumption with rate ν(Equation 2). People enter the consumption compartment with rate α and leave by becoming ill with rate τ (Equation 3). The infectious compartment holds all the people that become ill after consuming a tainted product (Equation 4). Contaminated food products can progress through the four stages consecutively to denote their place during a contamination event (i.e. total available products, susceptible, exposed, infectious). Contaminated food products are sent to different distribution channels (DC) (i.e. food retail, food service) for purchase and consumption. The contaminated food population is non-constant and homogeneous with a set amount of products available for purchase without rapid replenishment. The infectious compartment does not include shelf life. This case omits the recovered compartment due to the population being food.

2.2 Food Removal

2.2.1 Model Description and Assumption

The food removal case is the last adaptation of the SEIR models mentioned in chapter 2. Case 3 uses a deterministic ordinary differential equation progression model to illustrate the removal of contaminated food due to reported cases. The phase of progression is as follows: (1) The total population of contaminated food products is denoted by (A); (2) The number of contaminated food products purchased from a distribution channel that can potentially cause illness can be denoted as (P); (3) The number of food products that are not linked to illness are denoted as (C) ; (4) The number of food products that are linked to illness can be denoted as (I); and (5) The number of products that are reported as contaminated in order to activate a removal policy can be denoted as(R).This model is a variation of the original SEIR model because it considers the basic stages a contaminated food product takes to cause illness in a consumer. Based on the flow of each stage, we construct a system of differential equations (Figure 2).

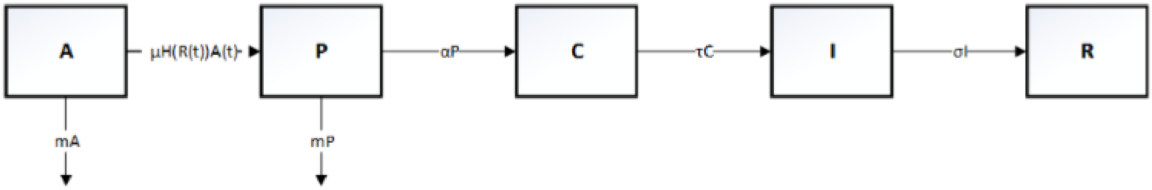

Figure 2. SEIR Model Flow Food Removal

Table 2 presents the parameters for the Case 2 model.

Table 2. Food Removal Model Notation

Symbol	Description
$A(t)$	The number of contaminated food products are available on shelf at time t
$P(t)$	The number of food products that are purchased from Distribution Channel (DC) at time t.
$C(t)$	The number of contaminated food products that are consumed at time t.
$I(t)$	The number of contaminated food product linked to illness at time t.
R(t)	The number of food products reported as case at time t.

Table 3. Food Removal Model Notation Cont.

Symbol	Description
m	The shelf life of the product.
μ	The rate which food products are purchased.
α	The rate which food is consumed.
τ	The rate at which a contaminated food product is linked to illness.
σ	The rate which a contaminated food product is reported as a case.
$H(R(t))$	Removal policy based on number of products that are linked to illness
δt	Time step (days)
s	Threshold for contaminated food products in order to issue a recall or intervention
t	Number of days in a food outbreak.

2.2.2 Model Formulation

$$\frac{dA}{dt} = -mA(t) - \mu H(R(T))A(t) \tag{5}$$

$$\frac{dP}{dt} = \mu H(R(t))A(t) - mP(t) - \alpha P(t) \tag{6}$$

$$\frac{dC}{dt} = \alpha P(t) - \tau C(t) \tag{7}$$

$$\frac{dI}{dt} = \tau C(t) - \sigma I(t) \tag{8}$$

$$\frac{dR}{dt} = \sigma I(t) \tag{9}$$

$$H\big(R(t)\big) = \begin{cases} 1 & R(t) \le s \\ 0 & otherwise \end{cases} \tag{10}$$

A finite number of products (A) are available for purchase. These products exit the compartment if they are purchased with rate μH(R(t)A(t)) and it reaches the shelf life m (Equation 5). Food products move to the purchase compartment with rate μH(R(t)A(t)). Food exits this compartment at a rate m based on the shelf life and with a rate α that a customer consumes the product (Equation 6). Food moves to the consumption compartment by purchases made at a retailer with rate α and ingestion with rate τ (Equation 7). After the customer consumes the product with rate τ, food exits the compartment when the product is linked to illness with rate σ (Equation 8). The removed compartment holds the number of products that are linked to illness (Equation 9). Based on the amount of food products links to certain threshold the flow is stopped to the retailer (Equation 10). Food products can progress through the five stages consecutively to denote their place during a contamination event (i.e susceptible, exposed, infective, removed). Contaminated food products are sent to different distribution channels (DC) (i.e. food retail, food service) to be purchased and consumed. When contaminated food causes illness it is reported. When the reported class reaches a certain threshold, products are removed from the shelf. The food population is considered non-constant and homogeneous due to the demand from consumer needs. This system of non-linear differential equations (Equations 5-10) can be solved using numerical approximation techniques.

3. Results

3.1 Case 1: Deterministic Differential Equation Model

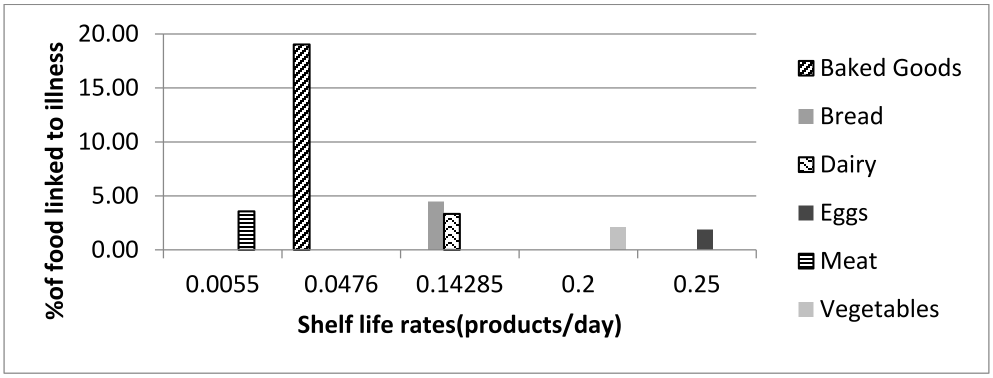

Figure 3. The percentage of food population linked to illness based on shelf life

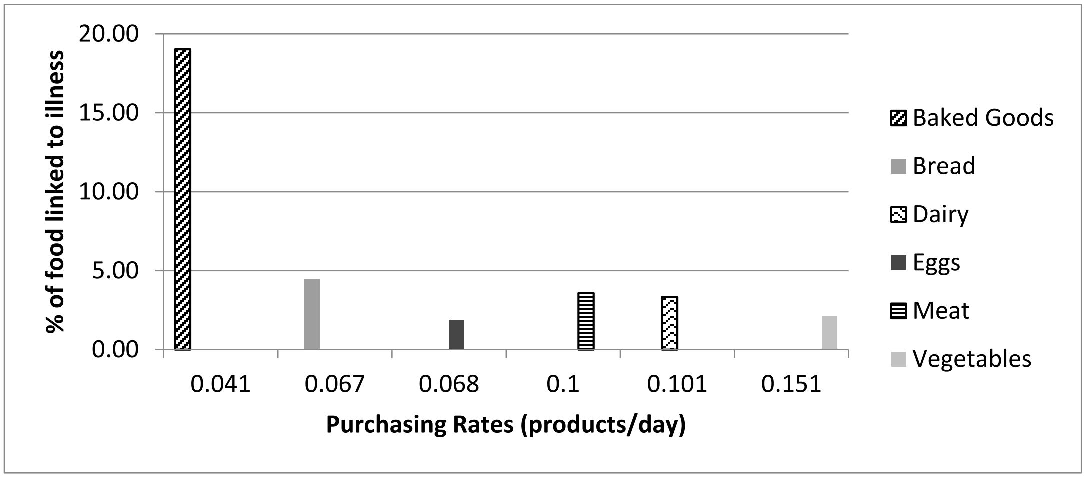

Figure 4. The percentage of food linked to illness based on consumption rate

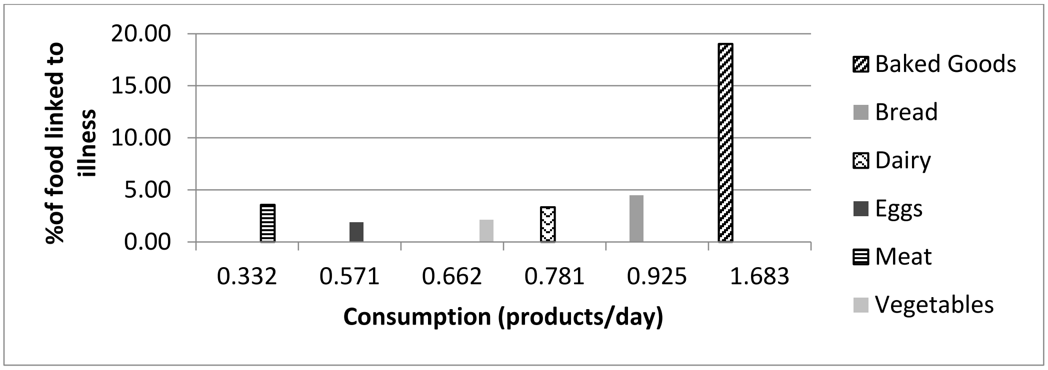

Figure 5. The percentage of food linked to illness based on consumption

The APCI model gives results for food products that are linked to illness based on shelf life, purchasing rates, and consumption rates. Meat has a shelf life of 180 days and results in the most amount of food products that are linked to illness (93.24%) (Figure 3). This is a result of consumers eating this food product at a slower rate (Figure 4). The long shelf life gives time for food safety specialists to determine when to recall food products and remove them from the shelf. However, eggs have the shortest shelf life and result in the least amount of food linked to illness with the lowest percentage (14.86%) of food linked to illness. Baked goods have the second longest shelf life of 21 days with results in (45%) of food products being linked to illness. Vegetables, dairy, and bread have (33.02%, 37.85% and 27.37%) respectively of food products linked to illness. This could be due to consumer behavior. Vegetables are purchased more often than the other high-risk food types (Figure 5). Since baked goods, have a longer shelf life these foods are purchased at a slower rate. Although each food type has various purchasing rates, companies decrease the spread of illness by issuing recalls.

3.2 Case 2: Food Removal

The APCIR model displays results for a supply chain intervention that is implemented to prevent contaminated dairy food products from remaining on the shelf for consumers to purchase. Figures 6 displays the results of omitting the shelf life associated with dairy as a representative case of the other food types which can be found in Appendix B. All high-risk foods reached the threshold of products that are needed to order a recall. Once this threshold is met, the available products remain constant over time meaning that people are not purchasing those items. When shelf life is considered with the recall process, it changes the dynamics of the food recall process. If the shelf life is longer, then it is easier to determine a food recall because you still have products on the shelf and consumers can be alerted (Figure 7). If the shelf life is smaller, there is a higher chance that the food will not be linked to illness immediately, but, products may already be consumed or removed from the shelf. As the amount of products reaches the threshold (4-6 days) of being linked to illness, a decrease occurs signifying an increase for products that are removed from the shelf. Determining the best time to remove products and also having the proper information and guidelines can help to keep contaminated food products away from consumers.

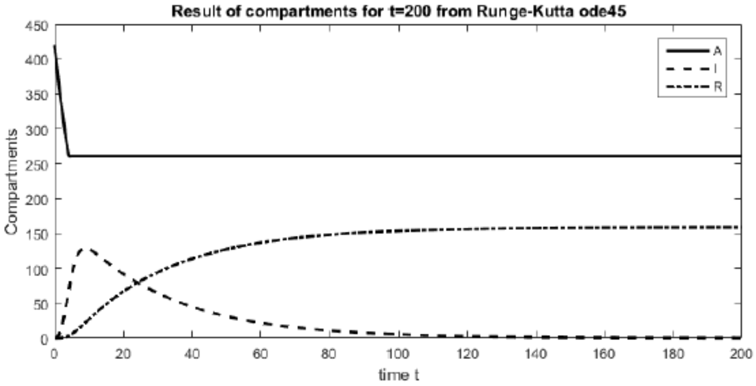

Figure 6. Dairy products intervention based on no shelf life

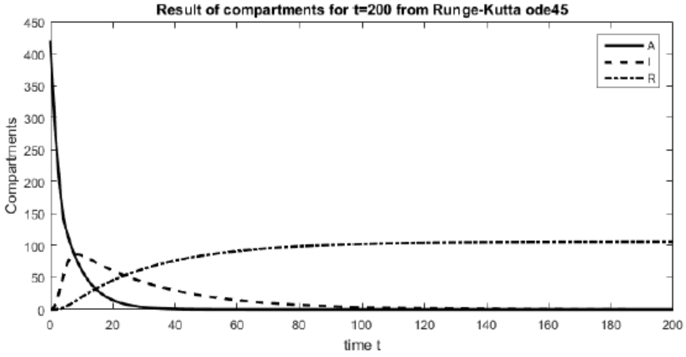

Figure 7. Dairy products intervention based on no shelf life

4. Conclusion

The compartmental models show the progression of food and human populations through various stages of contamination. Various purchasing rates, consumption rates, and shelf life data are used as parameters in these models to determine these measures. Based on the specific purchasing and consumption rates, some food products result in higher risk. The shelf life of food products has an impact on implementation of intervention strategies. A longer shelf life is conducive to a recall which results in food products being removed from various distribution channels and homes. Future work includes collecting more data for purchasing and consumption patterns from a diverse population can make results more robust.

5. References

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Talley, J., Davis, L. B., Morin, B., & Liu, L. (2019). A vector-borne contamination model to assess food-borne outbreak intervention strategies. *Applied Mathematical Modelling*, *66*, 383-403.