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Lot Size Optimization with RTI in a Closed-loop Supply Chain

P Thorat, Y Wang, and SW Yoon

State University of New York at Binghamton
Binghamton, NY 13902, USA

Corresponding author's Email: pthorat1@binghamton,edu

Author Note: Mr. Pravin Thorat is a Graduate Research Associate at Watson Institute for Systems Excellence, Binghamton University. Dr. Yong Wang is an Assistant Professor and Dr. Sang Won Yoon is an Associate Professor at the Department of Systems Science and Industrial Engineering, Binghamton University.

Abstract: The study in this paper shows a lot size optimization model with a returnable transport item (RTI) for the two-echelon closed-loop inventory system of purchased parts required in manufacturing. The system consists of one warehouse and one or more than one work center, in which purchased parts have an only forward direction of flow while RTI has forward as well as return flow. This two-echelon inventory system uses continuous-review (Q, r) ordering policy for warehouse and work center. To achieve an optimized inventory policy and a lot size of purchased parts, a mathematical model developed to minimize total inventory investment subject to a constraint on space at the work center, average order frequency and expected backorder level at the warehouse and work center. The model shown here is a pilot test for a broad set of purchased parts, and hence excel based solver used for preliminary results. A numerical example with pseudo data is provided to present the applicability of the proposed model and procedure. Finally, other possible extensions of the model used, and different possible solution methodologies discussed.

Keywords: Returnable Transport Items, Closed-Loop Supply Chain, Lot-Size, Supply Chain, Continuous Review Policy, Multi-Echelon Inventory

1. Introduction

Inventory management of purchased parts in a multi-echelon closed-loop environment with RTI to support the continuous operation of an assembly line of manufacturing with one or more levels of work center is a complex problem. An increase in operational reactiveness across the supply chain, improved service levels, and reduced inventory investment are some of the notable characteristics of successful inventory management. One of the essential aspects of closed-loop multi-echelon inventory management is the distribution of finished parts and the recovery of transport items back in the supply chain for repeated use.

The multi-echelon closed-loop supply chain systems are getting attention from both the company operation managers and academic researchers in various communication and industry network. For instance, motivation for this present study comes from observing and working in the present issues from one of the material handling equipment manufacturing company. To manufacture this equipment, the company has dedicated manufacturing and assembly lines, which requires continuous feeding of purchased parts at one or more than one distinct work center on the shop floor, to carry out a standard set of operation for completing the assembly within assigned takt time. Due to tight schedule and space constraints, the manufacturing team tries to keep minimum inventory at the work center, which satisfies the daily operation. The company offers a contract to suppliers for the supply of purchased parts and stock them in the warehouse, and from the warehouse, material handling team takes care of delivering the purchased parts to work centers. The company faces various challenges to keep balance in the work center requirement and total inventory investment. Also, the company has the plan to introduce RTIs and getting rid of all other one-time usable packaging items such as corrugated boxes, plastic bags, etc. by setting reorder quantity in multiple of RTIs quantity such way that, RTI weight should always remain within safety constraints for ease of operator.

2. Literature Review

Several types of research have been conducted in the closed-loop supply chain, RTIs, and multi-echelon inventory management systems. Research conducted by (Hariga, Glock, & Kim, 2016) on single-vendor single-buyer which takes into consideration of owned and rented option for RTIs with stochastic return time, problem formulated with the movement of final product and RTIs in the system to minimize the supply chain-wide cost and developed a solution procedure for optimal replenishment policy, the minimum number of RTIs and the minimum number of truck required for shipment. Also, a study was conducted to see the effect of rented and owned RTIs in the supply chain. (Cobb, 2016) developed a mathematical model for inventory management of RTIs in the closed-loop supply chain for used and newly purchased RTIs used in combination for production requirements, also giving an option for manager to select RTI inspection length and repair cycle length, based on which purchase cycle shown to a manager as output to a make purchase decision. Research with the capacity of RTIs carried out by (Glock & Kim, 2015) to assess the impact of the weight of RTIs in reorder quantity and overall supply chain and manage RTIs in such a way that the cost of producing the product and distributing with RTI is minimized. Research the two-echelon inventory system carried out by (Pasandideh, Niaki, & Tokhmehchi, 2011) for continuous review policy in which model formulated with a budget constraint on inventory and RTI investment. Although, the studies mentioned above are comparable to this study. Proposing an optimization model to determine the quantity of order, the quantity of RTIs, and when to order, such studies are in practice. The main contribution of this study is to develop a mathematical model by taking into account all practical aspects such as space constraints, safety weight constraints, and actual consumption at the work center that minimizes the total inventory investment in purchased parts and RTIs at the warehouse as well as at work centers.

3. Problem Definition

The company consists of a two-echelon supply chain of purchased parts with one central warehouse and various work centers in which all purchased parts replenished almost after each day. The system is observed to perform in the below manner: At the start of the cycle from the warehouse, a supplier sends a bulk lot of purchased parts to the warehouse. The warehouse team stock the purchased parts at various locations. The demand at the warehouse and work center is satisfied or back-ordered. At both stages of the supply chain, warehouse and work center inventory positions are continuously monitored by auditors and feed that data to the enterprise system, which triggers the reorder point, r and order of size, Q is placed either from the warehouse or work center. Figure. 1 represents a birds-eye view of the system under this study. To define mathematical problem following assumptions are considered in the model:

- The purchased parts supplier has an unlimited production capacity
- Backorders are allowed in warehouse and work center
- The supplier has a constant lead time
- The lead time to warehouse is fixed
- The number of order frequency at warehouse and work center are fixed as per company policy
- The backorders allowed at warehouse and work center are fixed as per company policy
- Number of working days for nine months cycle is 183 days
- Each working shift has 7.5 hrs.
- Safety weight constraint for RTI is 40 lbs
- Inventory at work center should not be more than one and half day’s consumptions, i.e., 11.25 hrs.

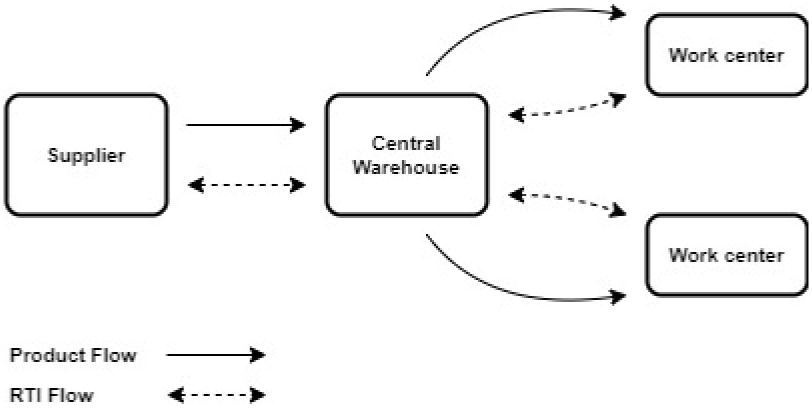

Figure 1. The two-echelon closed-loop supply chain system under study

4. Methodology

The mathematical problem was formulated to optimize lot sizes and total inventory at the warehouse and work center, subject to a limitation on space at the work center, order frequency, and backorder level. The parameters are as follows. The following notations, parameters, and variables are used for mathematical formulation of the problem:

j	Index for distinct parts
N	Number of distinct parts
m	Index for distinct work center
M	Number of distinct work centers
k	Index for RTI
D_j	$\sum_{m=1}^{M} D_{jm}$ Demand for part j at a warehouse
l_j	Lead time for part j at the warehouse
X_j	Demand for part j during a lead time at a warehouse
θ_j	$(D_j \times l_j)/183$ Expected demand during the lead time for part j at a warehouse
D_{jm}	Monthly demand for part j at work center m
l_{jm}	Lead time from warehouse to work center m for part j
X_{jm}	Demand for part j during the lead time at work center m
θ_{jm}	$(D_{jm} \times l_{jm})/183$ Expected demand during the lead time for part j at work center m
c_j	The unit cost of part j
k	Unit cost of RTI
w_j	Weight of part j in-lbs
w	Safety weight, 40lbs
Q_j	The order quantity for part j at a warehouse, this is decision variable
Q_{jm}	The order quantity for part j at work center m, this is decision variable
Q_{jk}	$\frac{(Q_j \times w_j)}{w}$ Quantity of RTI k for part j, this is decision variable
Q_{jkm}	$\frac{(Q_{jm} \times w_{jm})}{w}$ Quantity of RTI k for part j at work center m, this is decision variable
r_j	Re-order point for part j at a warehouse, this is decision variable
r_{jm}	Re-order point for part j at work center m, this is decision variable
h_j	Unit holding cost for part j at a warehouse (same as unit cost)
A	Unit purchase order cost
b	Unit backorder cost
$F_j(Q_j)$	Order frequency for part j at a warehouse
$F_{jm}(Q_{jm})$	Order frequency for part j at work center m
$B_j(Q_j, r_j)$	Backorder level for part j at a warehouse

$B_{jm}(Q_{jm}, r_{jm})$	Backorder level for part j at work center m
$S_j(Q_j, r_j)$	Service level for part j at a warehouse
$S_{jm}(Q_{jm}, r_{jm})$	Service level for part j at work center m
$I_j(Q_j, r_j)$	Inventory level for part j at a warehouse
$I_{jm}(Q_{jm}, r_{jm})$	Inventory level for part j at work center m
$I_{jk}(Q_{jk})$	Inventory level for RTI k at a warehouse
q_c	Purchased part consumption
h	Hours per shift, 7.5 hrs
d	Working days, 183 days
du	$\frac{q_c}{d \times h}$ Hourly Usage
F_{jT}	Order frequency allowed at a warehouse for part j
F_{jmT}	Order frequency allowed at work center m for part j
B_{jT}	Number of back-orders allowed at a warehouse for part j
B_{jmT}	Number of back-orders allowed at work center m for part j
TC	Total inventory investment

Now, the optimization model can be formulated as below:

Minimize:

$$TC = \frac{D_j}{Q_j} A_j + b_j B_j(Q_j, r_j) + b_{jm} B_{jm}(Q_{jm}, r_{jm}) + h_j I_j(Q_j, r_j) + h_{jm} I_{jm}(Q_{jm}, r_{jm}) + c_k I_{jk}(Q_j) + c_k I_{jmk}(Q_{jmk}) \tag{1}$$

Subject to:

$$F_j(Q_j) \le F_{jT} \tag{2}$$

$$F_{jm}(Q_{jm}) \le F_{jmT} \tag{3}$$

$$B_j(Q_j, r_j) \le B_{jT} \tag{4}$$

$$B_{jm}(Q_{jm}, r_{jm}) \le B_{jmT} \tag{5}$$

$$Q_j \ge 1 \tag{6}$$

$$Q_{jm} \ge 1 \tag{7}$$

$$Q_{jk} \ge 1 \tag{8}$$

$$Q_{jmk} \ge 1 \tag{9}$$

$$r_{jm} \ge du \times (l_{jm} + 1) \tag{10}$$

$$Q_{jm} \le du \times 11.25\ Hrs \tag{11}$$

$$w \le 40\ lbs \tag{12}$$

Eq. (1) is the objective function of the proposed model, which refers to both the echelons to give total inventory cost form purchased parts and RTIs. Constraints (2) and (3) make sure that order frequencies at both the echelons do not go beyond allowable values. Constraints (4) and (5) assures that backorder levels at both the echelons do not cross the allowable levels. Constraints (6) and (7) show that reorder quantities at a warehouse and work center are never zero. Constraint (8) and (9) restrict the number of RTI quantities always greater than or equal to one. Constraint (10) sets reorder point at work center such way that, purchased parts should remain in work center for consumption till next batch of Q_{jm} received. Constraint (11) make sure that reorder quantity at the work center do not cross one and half days of supply for consumption, to satisfy space constraint. Constraint (12) assures all RTIs should have a total weight of up to 40lbs for ease of picker.

The model formulated above is complex and difficult to solve exactly. A method like a branch and bound could be used for an optimum solution; however, it is not practical, as a set of purchased parts get large and large. In a backorder approach, all the exact values for reorder point are calculated based on demand probability as cumulative distribution function, service level $S_j(Q_j, r_j)$ and $S_{jm}(Q_{jm}, r_{jm})$, backorder level $B_j(Q_j, r_j)$ and $B_{jm}(Q_{jm}, r_{jm})$ and inventory level $I_j(Q_j, r_j)$ and $I_{jm}(Q_{jm}, r_{jm})$ are calculated from formulas given in (Hopp & Spearman, 2001) and (Svoronos & Zipkin, 1988). The solution methodology described in (Hopp & Spearman, 2001) is implemented here in an excel solver for a small set of purchased parts by setting desired order frequency and backorder levels. For a broader set of purchased part lists, different heuristic methods such as dynamic programming or genetic algorithm could be implemented.

5. Numerical Results and Analysis

To test formulated mathematical model numerical example is considered with the following data, and the model was run with an excel solver to get results. Data consist of the unit cost of a product, demand for a purchased part for nine months, lead time for warehouse and work center, for this example, only one work center is taken into consideration. To get the best solution mathematical model set up in an excel solver and after more than 110 iterations excel, solver spits out the preliminary result for below numerical example.

Table 1. Cost and demand data for multi-echelon, multi-part supply chain system

j	c_j	D_j	l_j	θ_j	σ_j	D_{jm}	l_{jm}	θ_{jm}	σ_{jm}
1	20.45	558	28	42.81	6.54	558	1	1.53	1.24
2	27.76	2077	28	159.33	12.62	2077	1	5.69	2.39
3	4.58	696	28	53.39	7.31	696	1	1.91	1.38
4	13.62	698	28	53.55	7.32	698	1	1.91	1.68

After complete iteration of excel solver for about 1 minute 12 sec on a system with 2.00 GHz Intel core 64-bit processor and 8GB RAM below results were obtained.

Table 2. Results for multi-echelon, multi-part supply chain system – iteration 1

j	Q_j	r_j	F_j	S_j	B_j	Q_{jm}	Q_{jk}	r_{jm}	F_{jm}	S_{jm}	B_{jm}	I	I_{Total}
1	46	47	12	0.97	8	17	1	56	33	0.99	0	1131.18	
2	102	176	20	0.99	36	7	1	114	296	0.37	2	3074.66	
3	77	99	9	0.96	2	40	1	111	17	0.95	1	579.85	
4	58	72	12	0.96	6	23	1	100	30	0.99	0	1133.48	
Total/Avg			13	0.97	52				94	0.83	3	5919.17	
Cost			416		1560				1974		30	3980	9899.17

Results in Table 2 obtained for allowable backorders at the warehouse set to 52 and the work center to 18 for this order frequency at the warehouse obtained to 13 and work center to 94. High frequency of order at work center is a result of space constraints as not more than one and half days of supply could accommodate at work center; hence to satisfy daily operations, a number of orders are received from the work center to warehouse is on the higher side. Also, from the result table, we can see that part number 2 service level at the work center is reduced due to many orders, which also shows a higher reorder point. However, the average service level at the work center is 0.83 for combined, which is satisfactory. To improve the service level best possible recommendation to increase the work center capacity to accommodate a large number of purchased parts.

Table 3. Results for multi-echelon, multi-part supply chain system – iteration 2

j	Q_j	r_j	F_j	S_j	B_j	Q_{jm}	Q_{jk}	r_{jm}	F_{jm}	S_{jm}	B_{jm}	I	I_{Total}
1	47	35	12	0.81	11	17	1	56	33	0.99	0	971.20	
2	108	135	19	0.76	51	7	1	113	297	0.37	1	2426.43	
3	80	83	9	0.99	3	40	1	111	17	0.95	0	519.86	
4	61	57	11	0.97	10	23	1	99	30	0.99	0	994.04	
Total/Avg			13	0.88	75				95	0.83	1	4911.53	
Cost			416		2250				1995		10	4671	9582.53

Some key managerial insights and conclusions are as follows. First, the fixed lead time from warehouse to work center may not maintain, however the optimized re-order point and re-order quantity with space constraint make the supply chain standardize the operation to achieve fixed lead time with minimum inventory. Also, the required number of RTIs speed up the time for material handling from the warehouse to the work center as redundant part movement and part counting is avoided. Second, by knowing the number of RTI requirements for each purchased part, the cost estimation of investment in RTIs is known, which may be less than the combined labor cost required for part picking from the warehouse without RTIs annually. The practical implication of this model is that this model requires some input such as safety weight constraints, purchase part consumption at the work center, and space constraint due to landlock situation. To obtain this data, the model guides the company indirectly to make all entities of the supply chain in alignment, starting from the supplier to the work center.

Total of three iterations carried out with different backorder levels and plotted in Fig. 2 graphs of total inventory investment vs. service level for the order of frequency at the warehouse, $F_j = 13$.

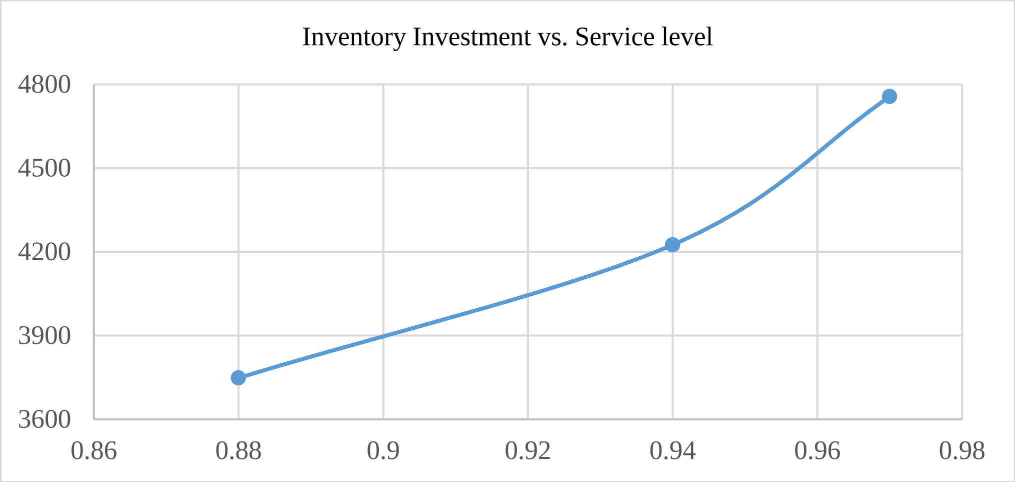

Figure 1. The two-echelon closed-loop supply chain system under study

6. Conclusion and Future Work

In this study, a mathematical model developed for lot size optimization considering space constraints, safety weight constraints, and actual consumption at the work center that minimizes the total inventory investment in purchased parts and RTIs at the warehouse and work center. We explored the use of daily consumption rates at the work center to limit excessive inventory at the warehouse and work center and solved this problem with excel solver. This model is more applicable to real-world purchased part inventory problems in the closed-loop supply chain. Future research may consider a machine learning approach to solve the problem considering historical inventory consumption at the work center.

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